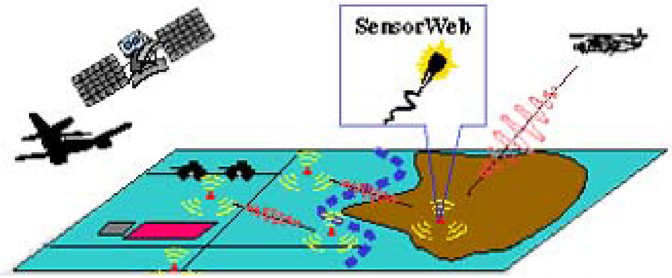

Optimization-based Approach to Source Localization and Self-Calibration in Distributed Arrays

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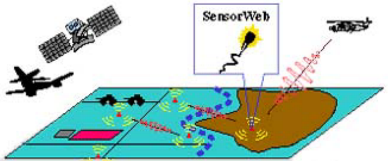
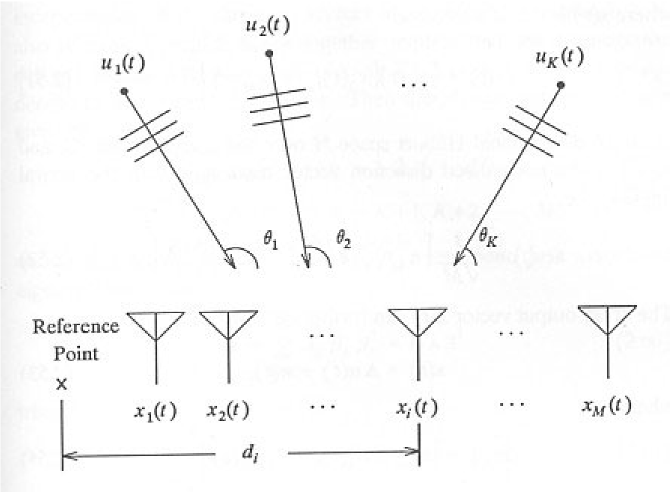
Stochastic Systems Group, M.I.T.

SensorWeb MURI Review Meeting

June 14, 2002



Source Localization Goals



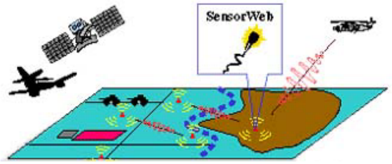
Context:

- Acoustic sensors
- Narrowband/broadband signals
- Far-field/near-field sources
- Any array configuration

Objectives for the new approach:

- Superior source localization performance (e.g. resolution)
- Robustness to limitations in data quality or quantity
- Self-calibration capability to handle uncertainties in sensor locations

Why is this interesting? How do we solve it?

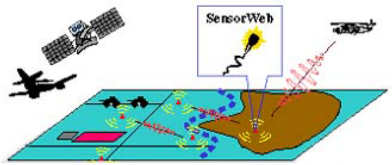


<u>Relevance for the SensorWeb context:</u>

- Limited aperture → limited Rayleigh resolution
- Limited observation time, low SNR
- Sensor locations known only approximately

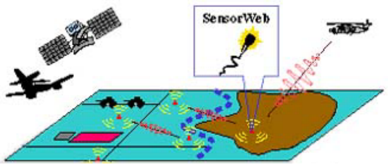
<u>Proposed approach:</u>

- View the problem as one of *imaging* a "source density" over the field of regard
  - Ill-posed inverse problem
  - Cast as an optimization problem and *regularize* by favoring fields with *concentrated densities*
  - Include optimization over sensor locations



Vital Statistics

- IT-2 (Fusion of heterogeneous sensors in unstructured and uncertain environments)
- RCA-1 (Self-calibration)
- Ties to RCA-2&3 (Tradeoffs in local vs. global processing) and RCA-4 (Minimum resource requirements)
- Contributors
  - Malioutov, Çetin, Fisher, Willsky
- Preliminary outputs
  - Several publications and talks
  - A number of academic, industrial, and DoD interactions



Preliminaries

- Consider M sensors, K source signals $u_k(t)$

$$g_m(t) = \sum_{k=1}^{K} u_k(t - \tau_m(\theta_k)) + n_m(t)$$

Source location parameters

Observations at the m-th sensor

Time delay to the m-th sensor

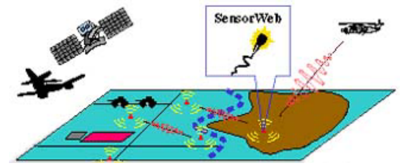
Noise at the m-th sensor

- Time delay structure depends on far vs. near-field
- In frequency domain (combining all sensors):

$$\mathbf{g}(\omega) = \mathbf{A}(\omega, \Theta)\mathbf{u}(\omega) + \mathbf{n}(\omega)$$

where $\mathbf{A}_{mk}(\omega, \Theta) = \exp(-j\omega\tau_m(\theta_k))$

- Note $\mathbf{A}(\omega, \Theta)$ depends on actual source locations



Observation Model

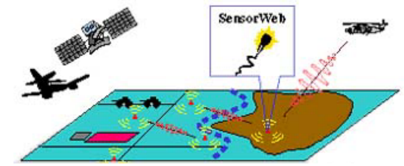
- Let $\{\beta_1, ..., \beta_{N_\beta}\}$ be a sampling grid of all source locations
- Define a $N_\beta \times 1$ vector $\mathbf{s}(\omega)$

$$\mathbf{s}_i(\omega) = \begin{cases} u_k(\omega), & \text{if } \beta_i = \theta_k \\ 0, & \text{otherwise} \end{cases}$$

- Define the $M \times N_\beta$ steering matrix $\mathbf{A}(\omega)$ (linking all potential source locations to all sensors)
- Resulting "overcomplete" observation model:

$$\mathbf{g}(\omega) = \mathbf{A}(\omega)\mathbf{s}(\omega) + \mathbf{n}(\omega)$$

- Formulate as a sparse signal reconstruction problem
- Determine source locations from peaks in reconstructed signal energy



A Variational Framework for Source Localization

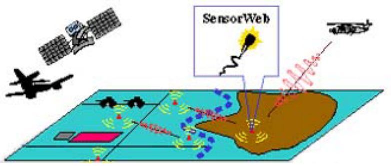
- Minimize the cost function:

$$J(\mathbf{s}) = \|\mathbf{g} - \mathbf{A}\mathbf{s}\|_2^2 + \alpha\|\mathbf{s}\|_p^p$$

Data fidelity

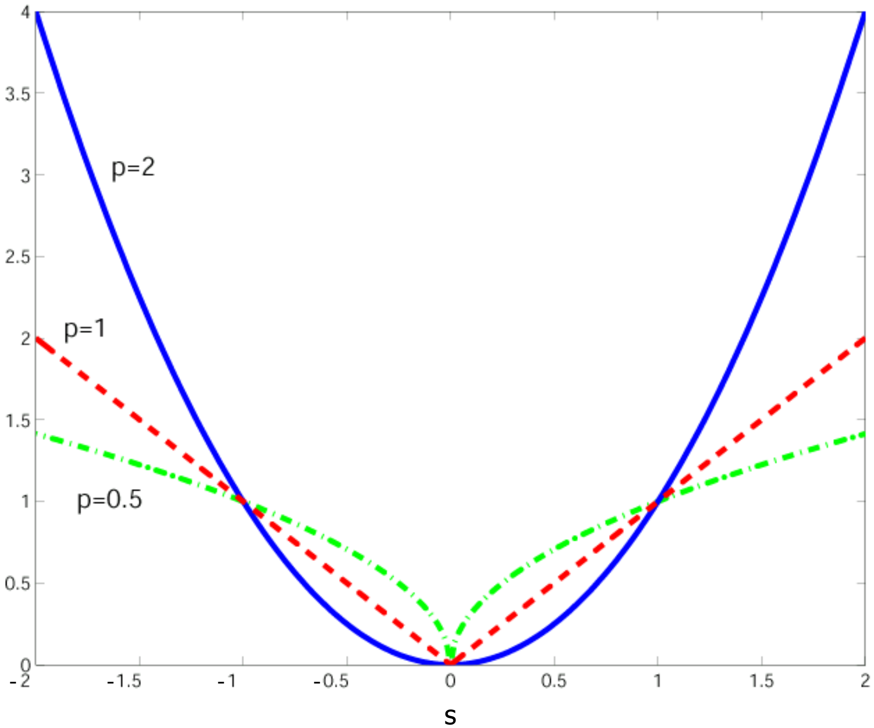
Regularizing sparsity constraint

- Role of the regularizing constraint $(p \leq 1)$:
  - Preservation of strong features (source densities)
  - Preference of sparse source density field
  - Can resolve closely spaced radiating sources
  - Other non-quadratic functions can be used

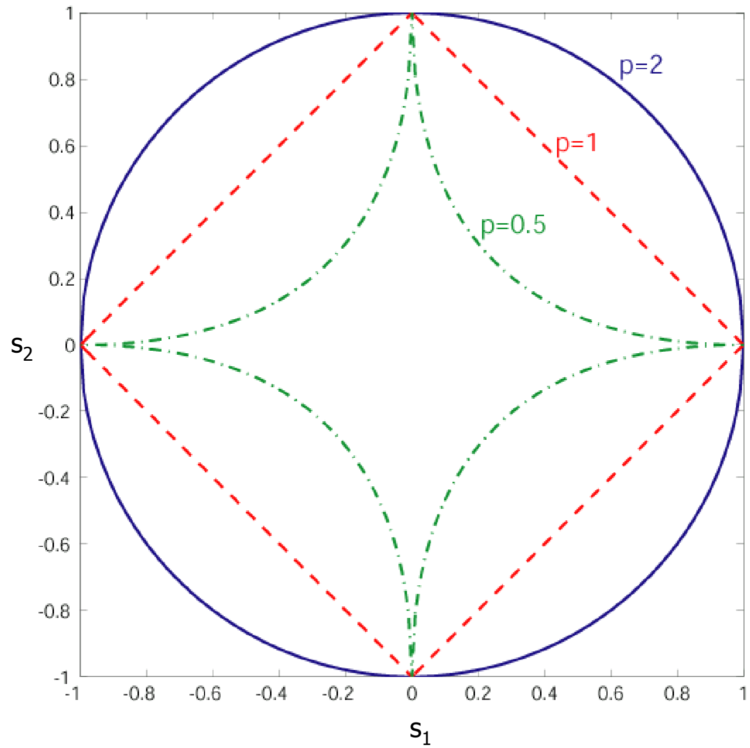


Expected Impact of Regularizing Constraints

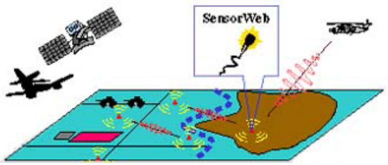
$|s|^p$



Level sets of $||\mathbf{s}||_p^p$



Using a relatively small p in the minimization of the ℓ_p-norm of a vector results in the preference of a sparser vector structure



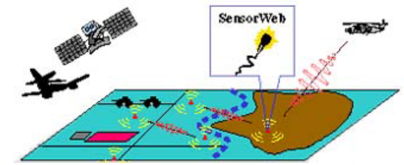
Solution of the Optimization Problem

- Cost function (differentiable approximation):

$$J(\mathbf{s}) = ||\mathbf{g} - \mathbf{A}\mathbf{s}||_2^2 + \alpha \sum_{i=1}^{N_\beta} (|\mathbf{s}_i|^2 + \epsilon)^{p/2}$$

- Gradient of the cost function:

$$\nabla J(\mathbf{s}) = 2\left(\mathbf{H}(\mathbf{s})\ \mathbf{s} - \mathbf{A}^H \mathbf{g}\right)$$



Solution of the Optimization Problem

- Iterative Scheme:

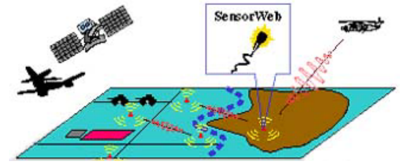
$$\mathbf{H}\left(\widehat{\mathbf{s}}^{(n)}\right)\ \widehat{\mathbf{s}}^{(n+1)} = \mathbf{A}^H \mathbf{g}$$

where n denotes the iteration number, and

$$\mathbf{H}(\mathbf{s}) \triangleq \mathbf{A}^H\mathbf{A} + \alpha\mathbf{\Lambda}(\mathbf{s})$$

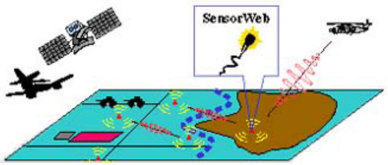
$$\mathbf{\Lambda}(\mathbf{s}) \triangleq \text{diag}\left\{\frac{p/2}{(|\mathbf{s}_i|^2+\epsilon)^{1-p/2}}\right\}$$

- Can be interpreted as a Quasi-Newton method with Hessian approximation $2 \cdot \mathbf{H}(\cdot)$ and unit step size
- Each step solves a quadratic optimization problem with intuitive, spatially adaptive weights

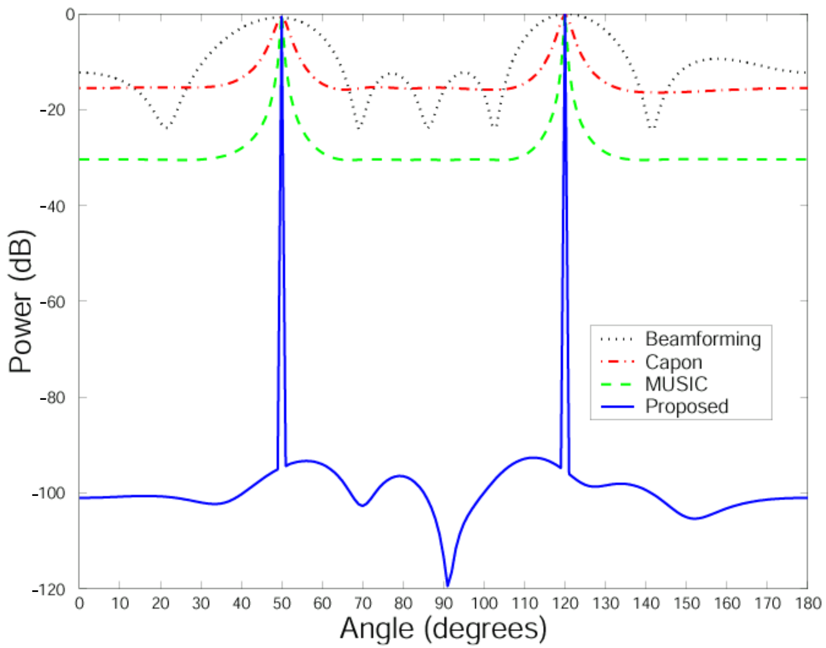


Overview of Experiments

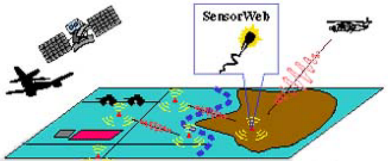
- Narrowband, far-field
  - Performance analysis based on multiple trials as a function of SNR and number of snapshots
- Narrowband, near-field
- Broadband, far-field
- Linear, circular, cross, rectangular arrays
- 200 time samples
- Use $p = 0.1$ in our objective function
- Choose α by subjective assessment



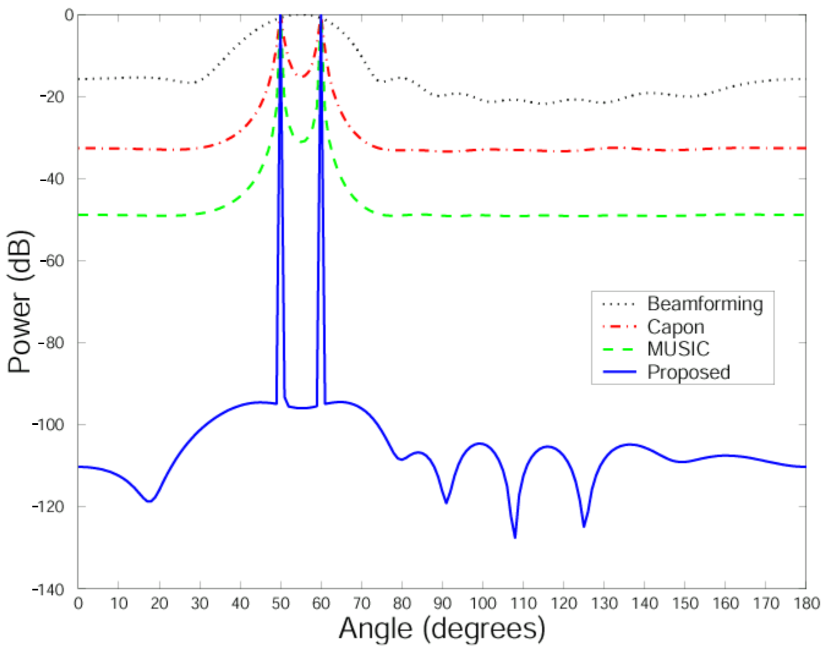
Narrowband, far-field



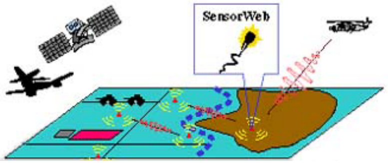
- Uniform linear array with 8 sensors
- Uncorrelated sources
- DOAs: 50°, 120°
- SNR = 10 dB



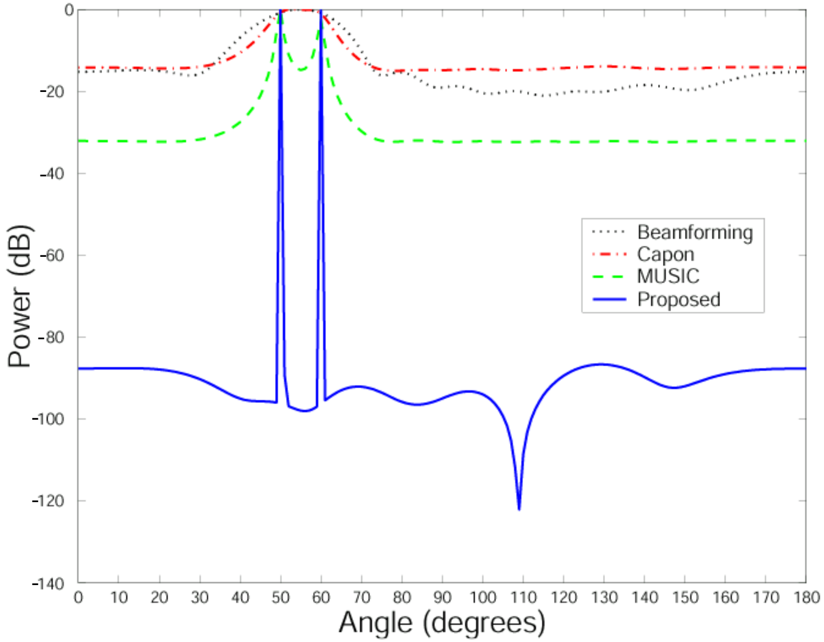
Narrowband, far-field



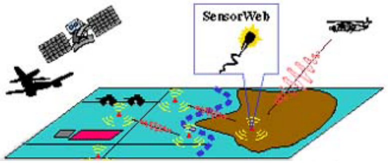
- Uniform linear array with 8 sensors
- Uncorrelated sources
- DOAs: 50°, 60°
- SNR = 20 dB



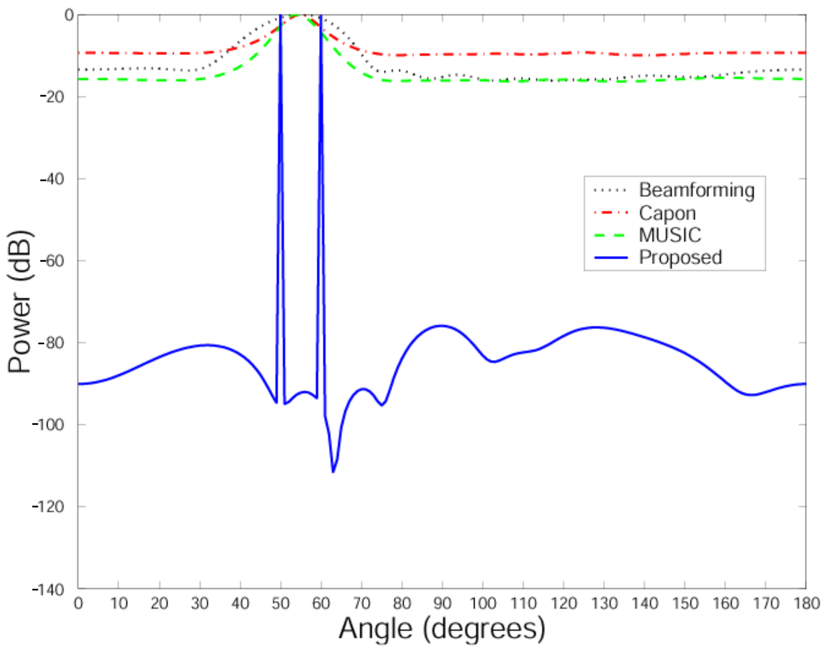
Narrowband, far-field



- Uniform linear array with 8 sensors
- Uncorrelated sources
- DOAs: 50°, 60°
- SNR = 10 dB

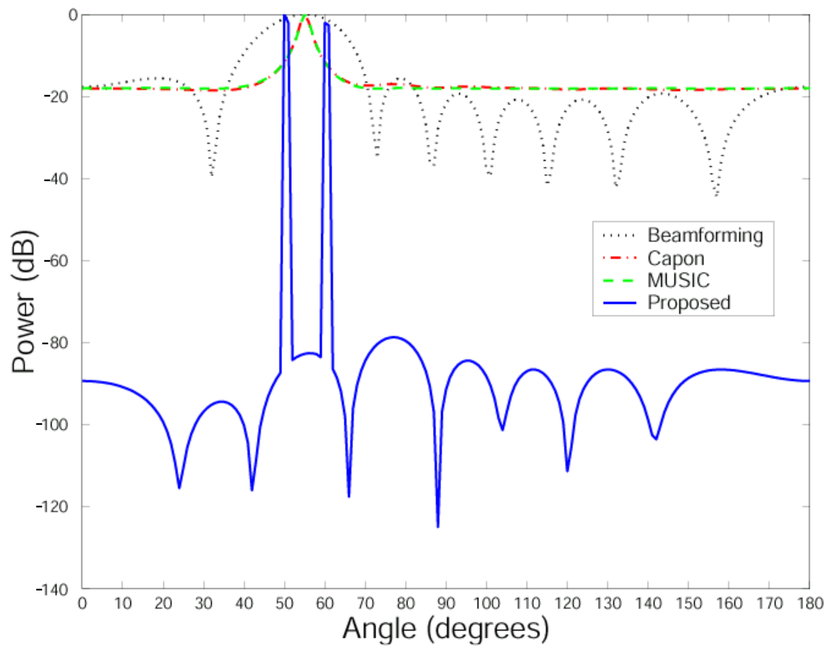


Narrowband, far-field

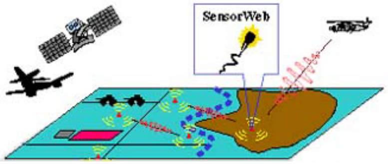


- Uniform linear array with 8 sensors
- Uncorrelated sources
- DOAs: 50°, 60°
- SNR = 5 dB

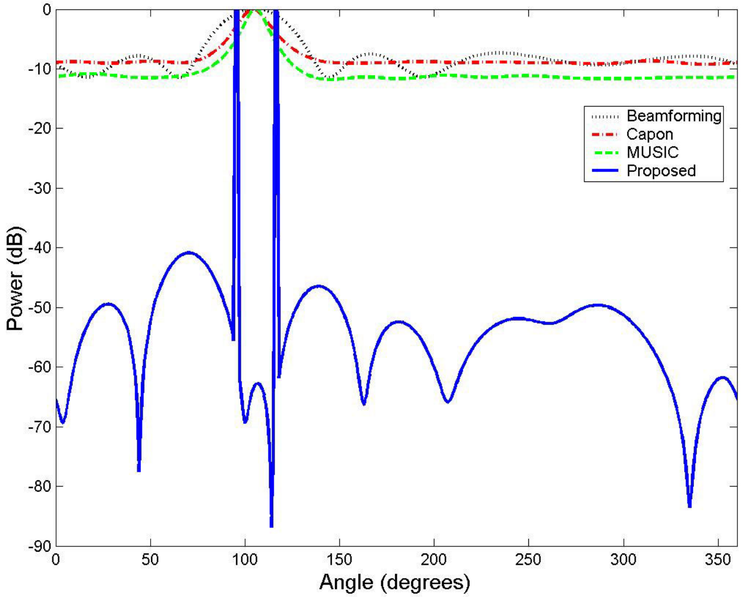
Narrowband, far-field



- Uniform linear array with 8 sensors
- Coherent sources (e.g. due to multipath)
- DOAs: 50°, 60°
- SNR = 20 dB

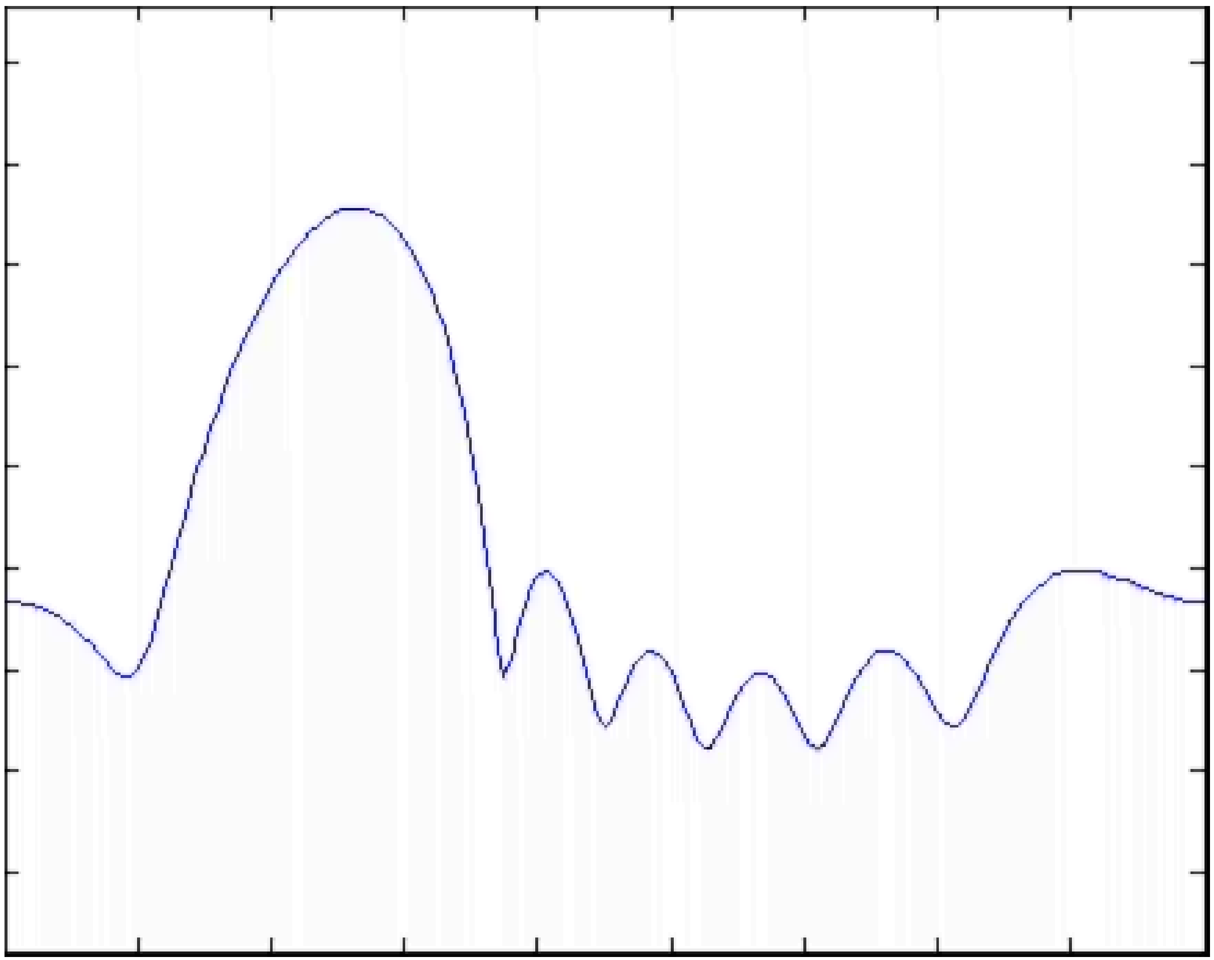


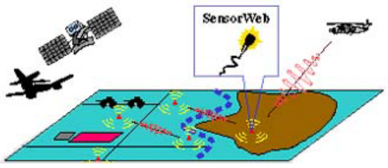
Narrowband, far-field



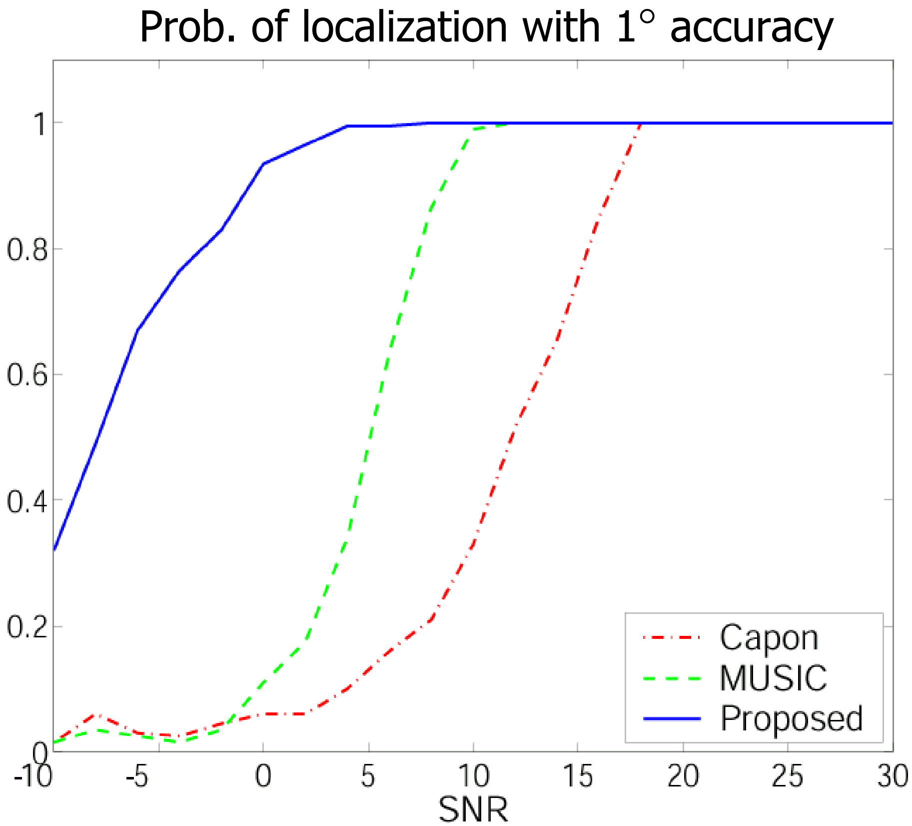
- Circular array with 10 sensors
- Uncorrelated sources
- DOAs: 90°, 120°
- SNR= 10 dB

Iterative behavior

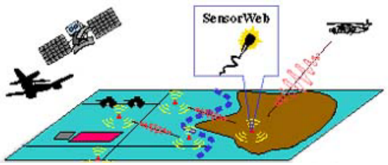




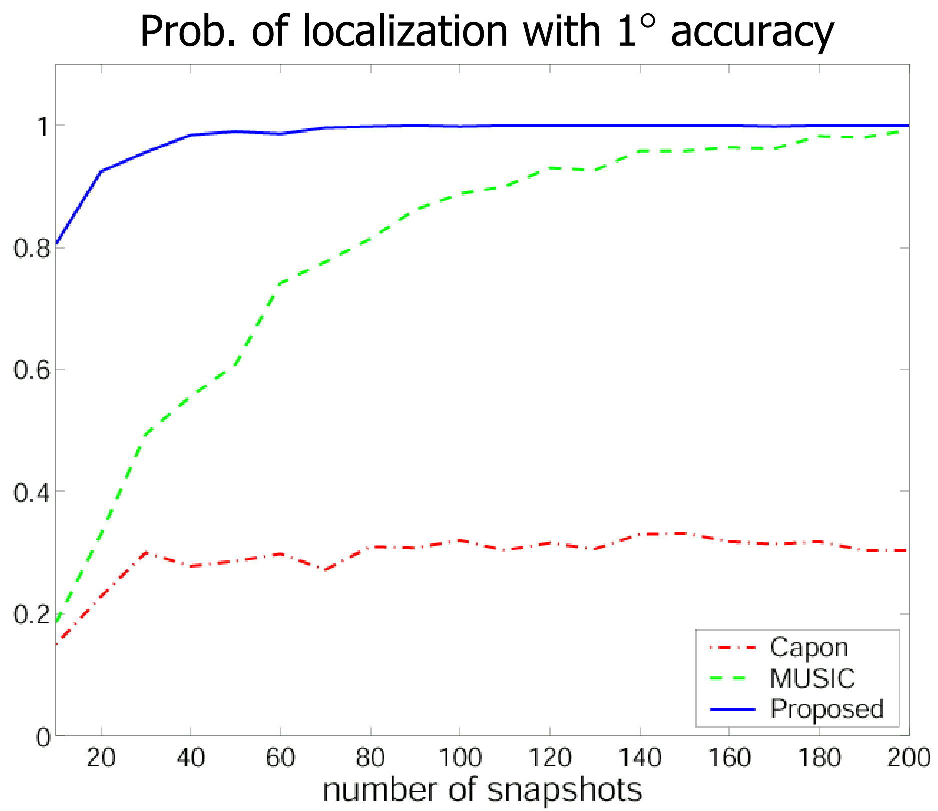
Prob. Correct Localization vs. SNR



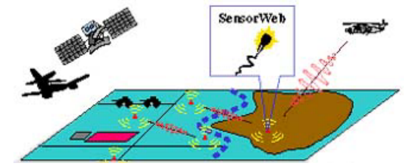
- DOAs: 50°, 65°
- Number of independent trials = 200
- Have similar results based on RMSE



Prob. Correct Localization vs. # snapshots

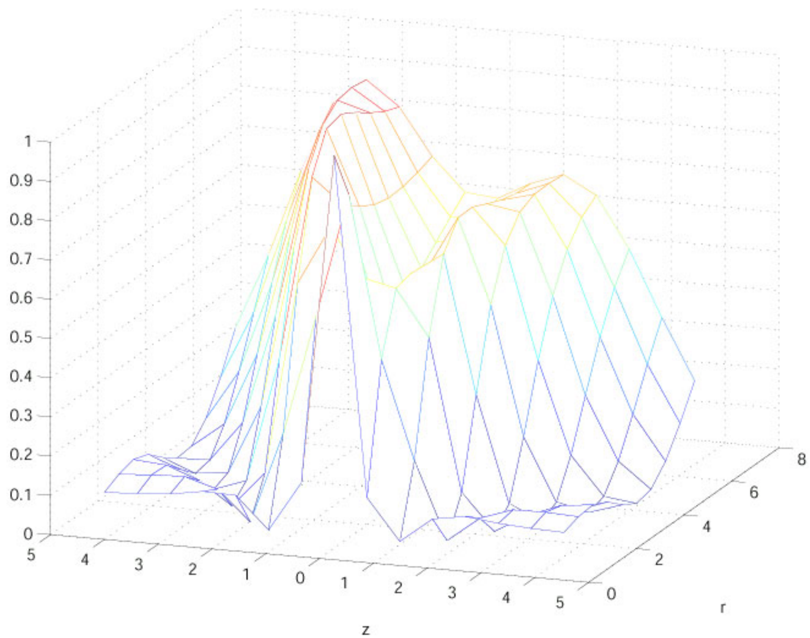


- DOAs: 50°, 65°. SNR = 10 dB.
- Number of independent trials = 200

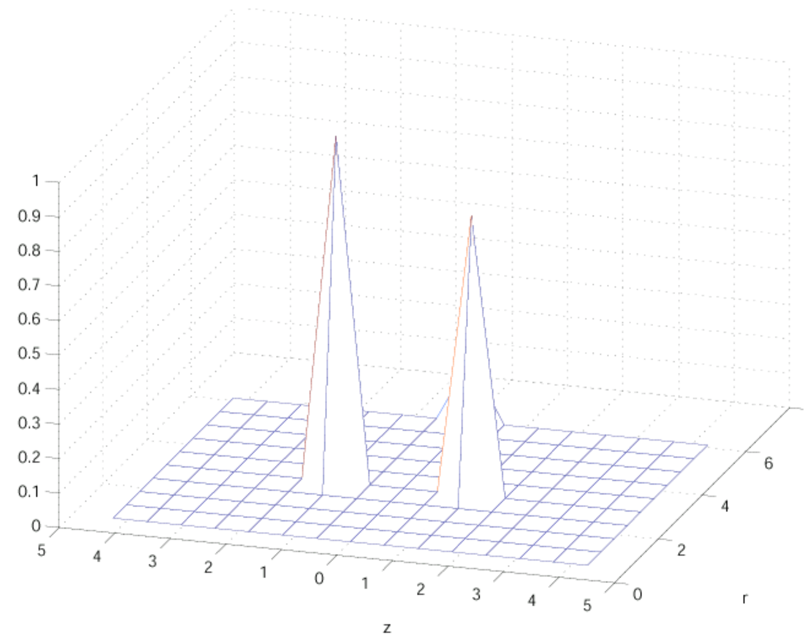


Near-field

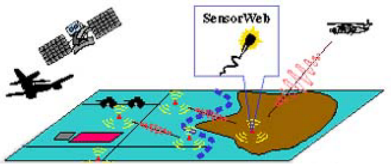
Conventional beamforming



Proposed

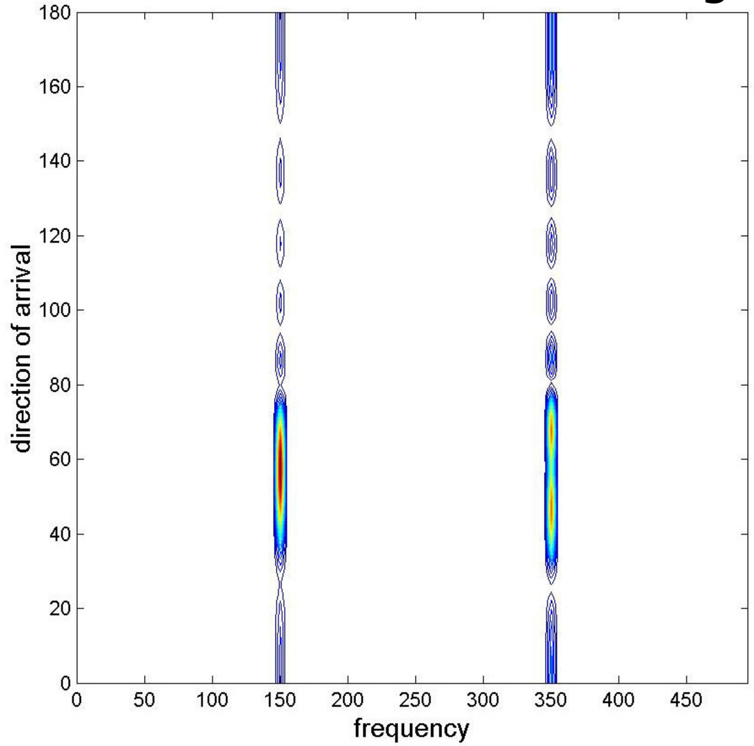


- Uniform linear array with 8 sensors
- Two uncorrelated sources

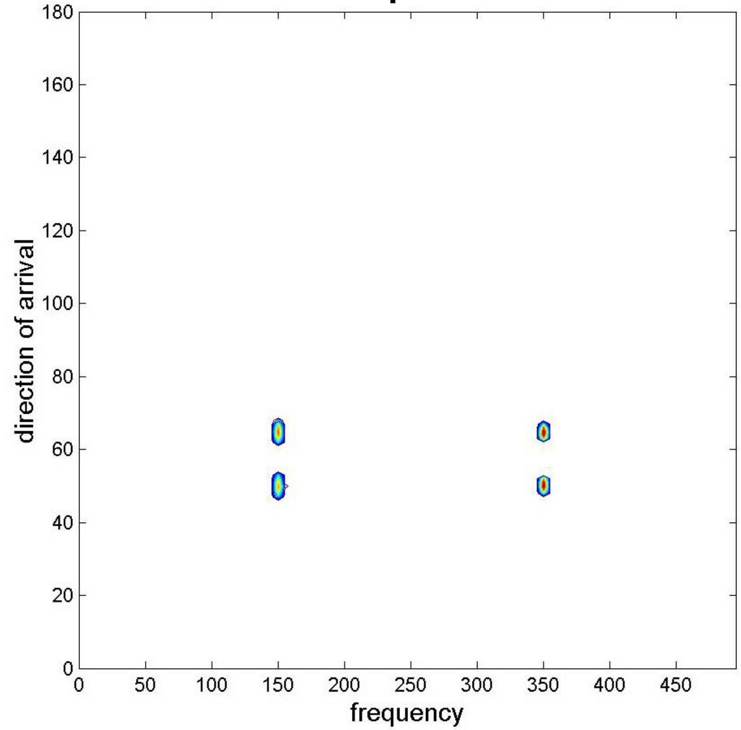


Multiple harmonics

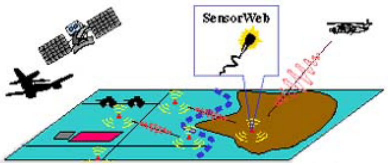
Conventional beamforming



Proposed

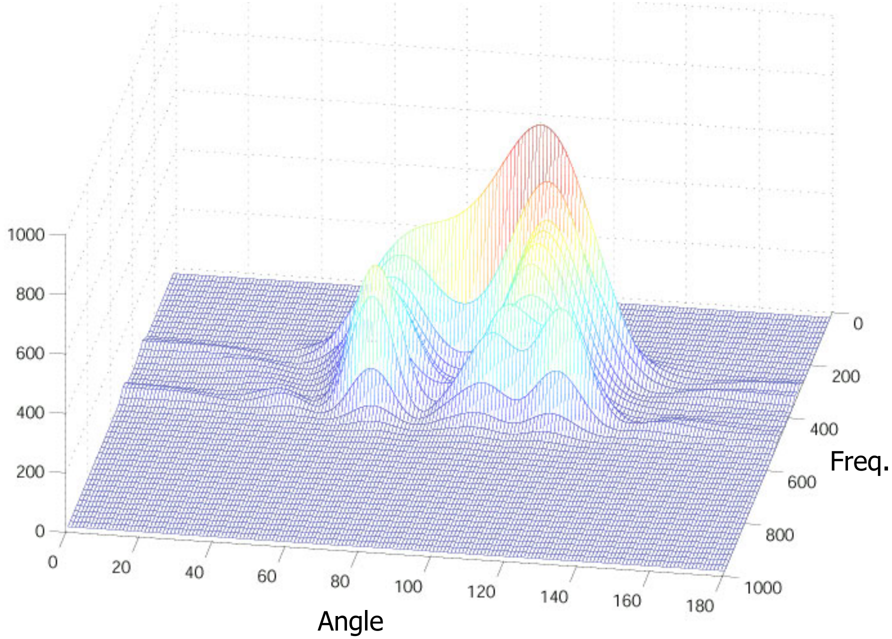


- Harmonics at 150 & 350 Hz, with DOAs: 50°, 65°
- SNR = 30 dB, uniform linear array with 8 sensors

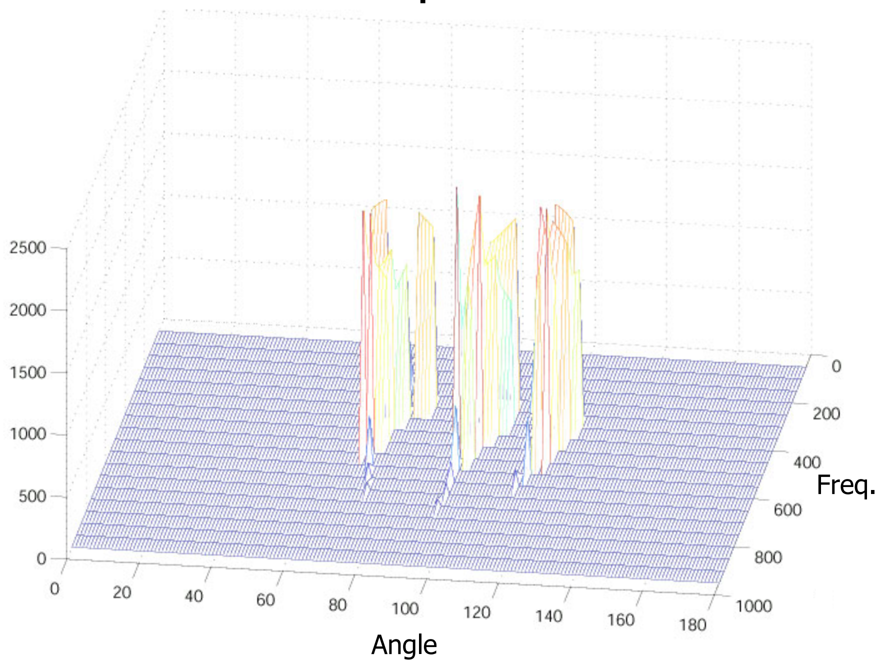


Broadband

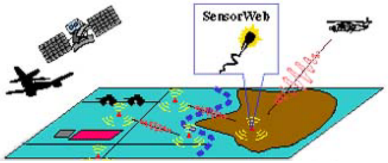
Conventional beamforming



Proposed



- Three chirp signals



Extension to self-calibration

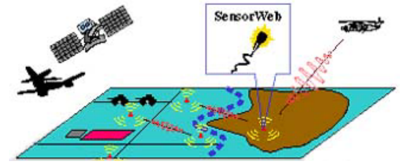
- Preliminary approach

$$J(\mathbf{s},\mathbf{r}) = \|\mathbf{g} - \mathbf{A}(\mathbf{r})\mathbf{s}\|_2^2 + \alpha\|\mathbf{s}\|_p^p$$

Sensor locations

- Use block coordinate descent for optimization

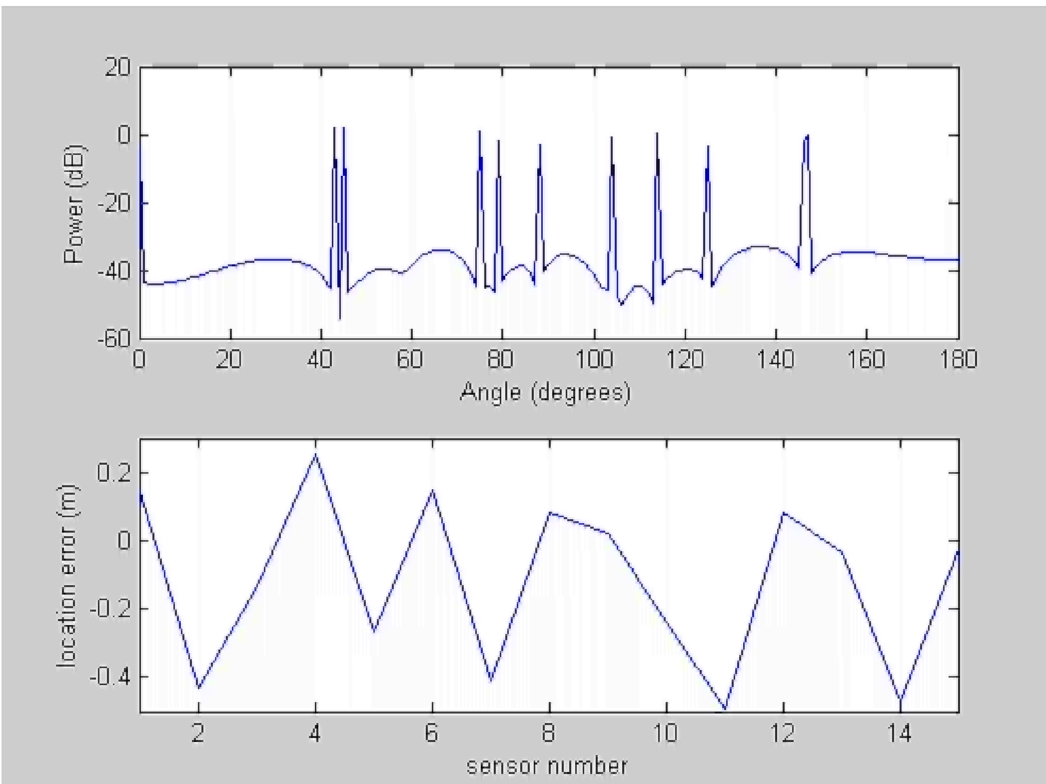
$$\begin{aligned}\hat{\mathbf{s}}^{(n+1)} &= \arg\min_{\mathbf{s}} J(\mathbf{s}, \hat{\mathbf{r}}^{(n)}) \\ \hat{\mathbf{r}}^{(n+1)} &= \arg\min_{\mathbf{r}} J(\hat{\mathbf{s}}^{(n+1)}, \mathbf{r})\end{aligned}$$

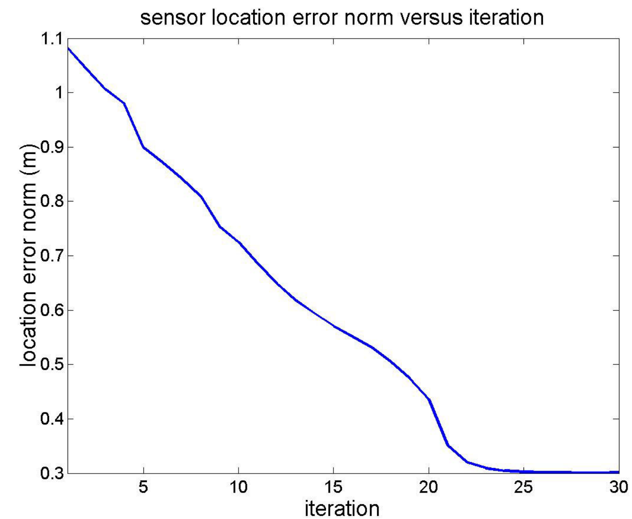


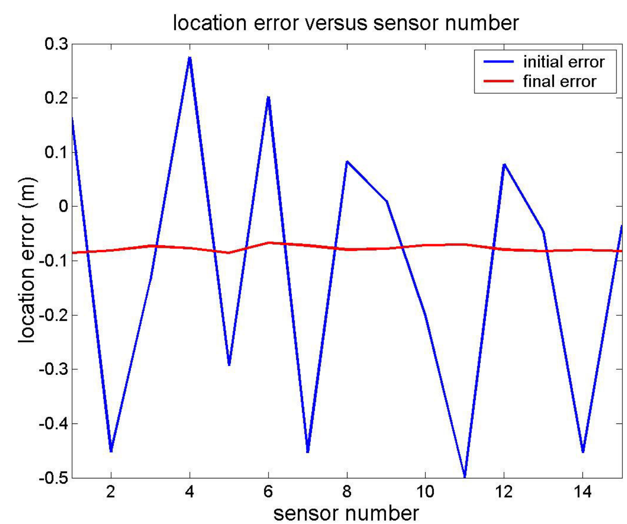
Self-calibration experiments

- Setup:
  - Far-field case
  - Narrowband signals
  - Linear array with 15 sensors
  - Two uncorrelated sources
  - DOAs: 45°, 75°
  - SNR = 30 dB
  - Sensor locations perturbed with a standard deviation of 1/3 of the nominal sensor spacing
  - 2-D array experiments underway

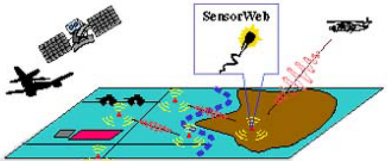
Self-calibration experiments – I





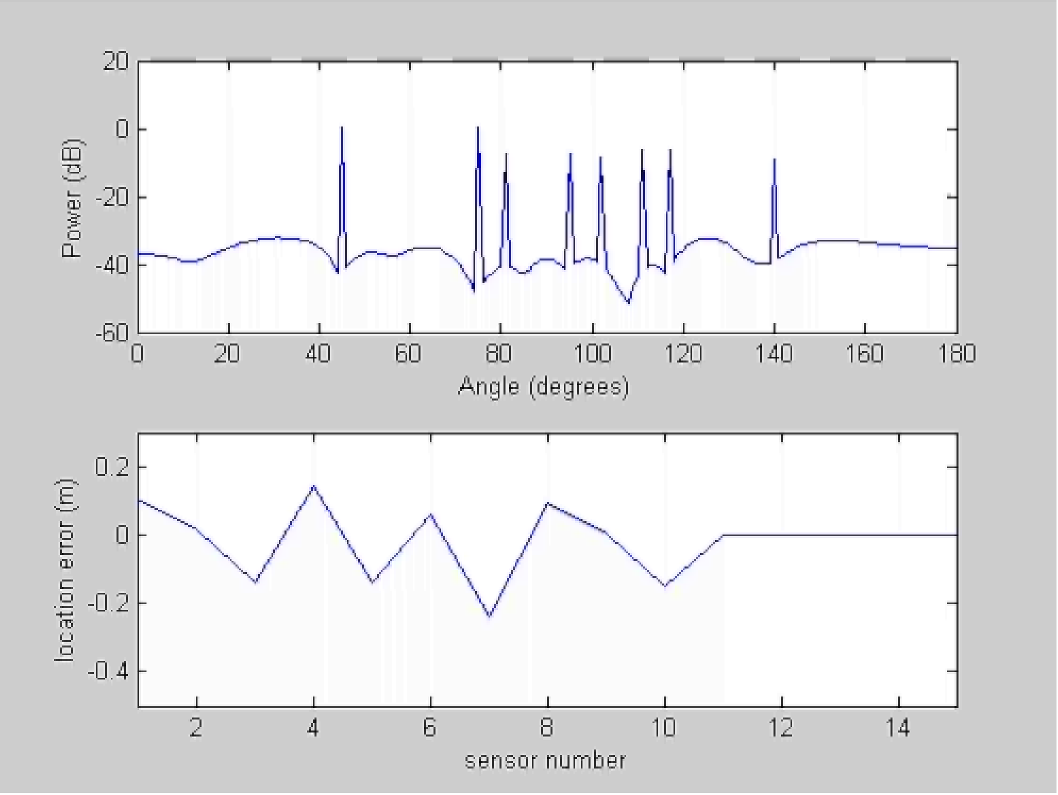


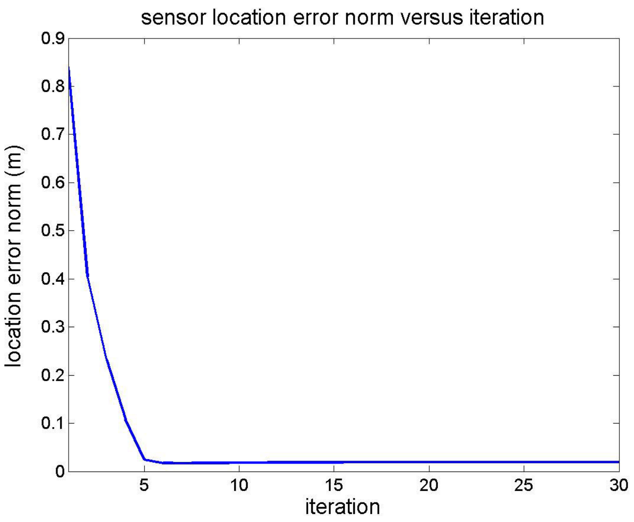
Moderate calibration errors can be compensated up to intrinsic ambiguities

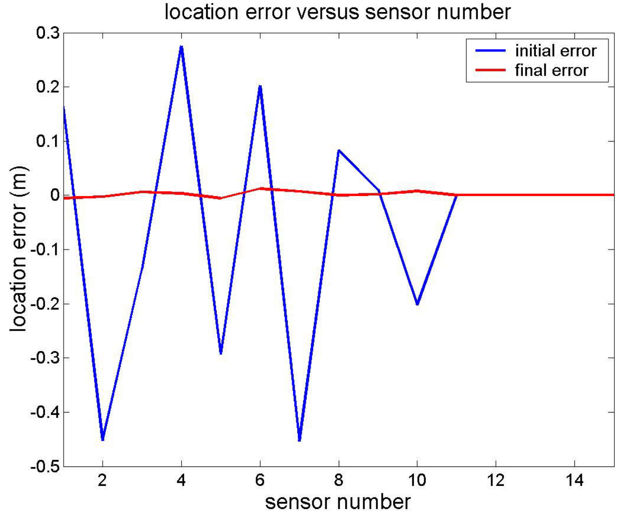


Self-calibration experiments – II

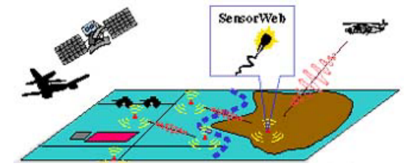
Some sensor locations known



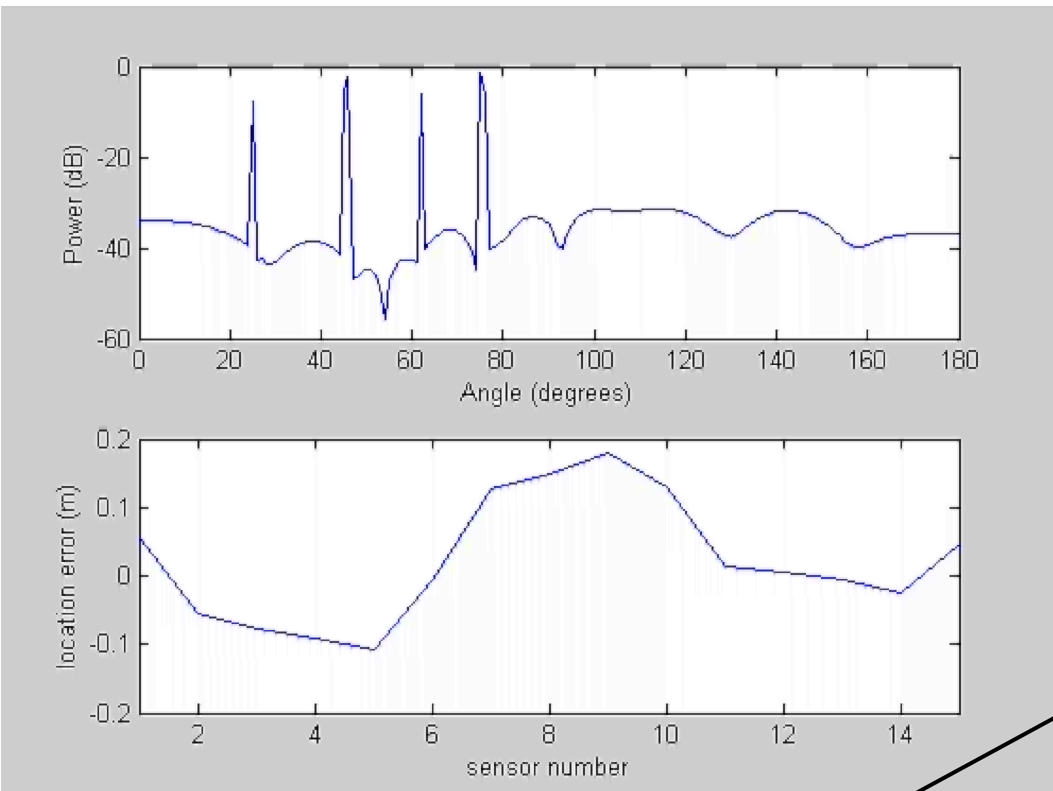


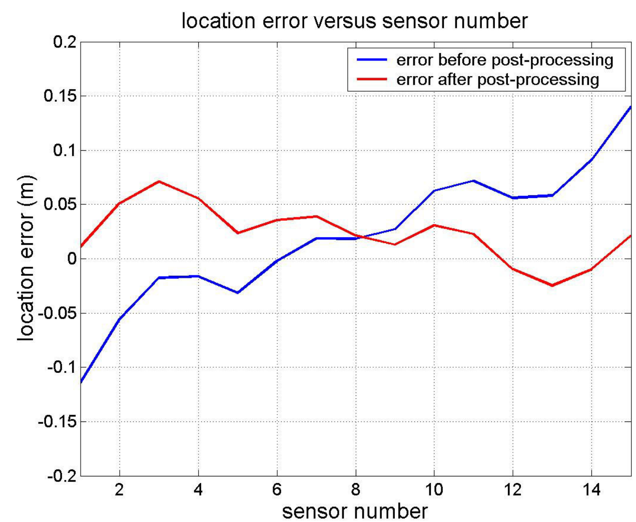


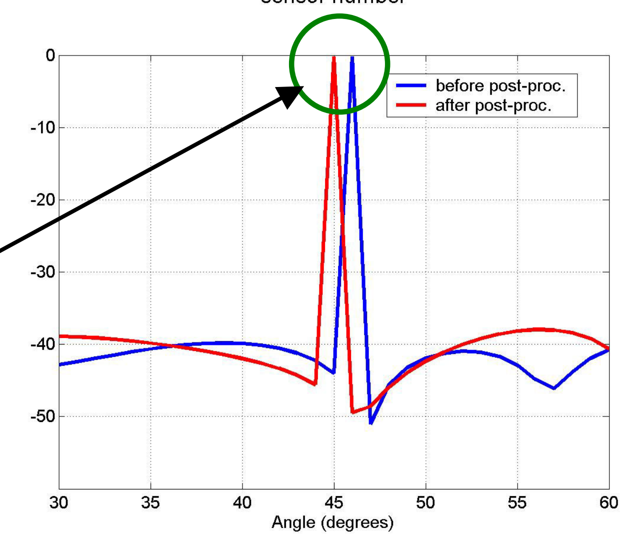
Additional information can be used to resolve the ambiguities



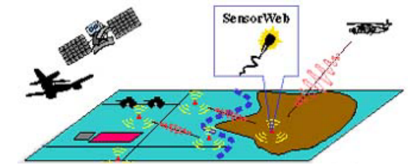
Self-calibration experiments – III





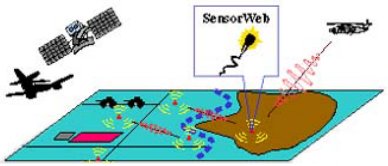


Side information can be used to discover and correct structural ambiguities



Summary

- Regularization-based framework for source localization with passive sensor arrays
  - Superior source localization performance
    - Superresolution
    - Reduced artifacts
  - Robustness to resource limitations
    - SNR
    - Observation time
    - Available aperture
  - Self-calibration capability
    - Can handle moderate uncertainties in sensor locations



Current and Future Work

- More on self-calibration
  - Gain/phase uncertainties in sensors
  - Ties to “autofocusing” methods in other domains
  - Identify limits on how much calibration error can be tolerated
  - Multiple arrays, complementary ties to Moses/Srour
  - Apply to the spatial coherence loss problem
- Experiments with measured data
- Issues to investigate
  - Choice of regularizing functionals and hyperparameters
  - Analysis of statistical performance, bounds
  - Tradeoffs between relatively local vs. global processing
- Extensions
  - Mobile/non-stationary environments
  - Heterogeneous sensors
  - Complex media
  - Directional sensors

Plans for measured-data experiments